Risk Analysis in Communication Networks with Conditional Value-at-Risk

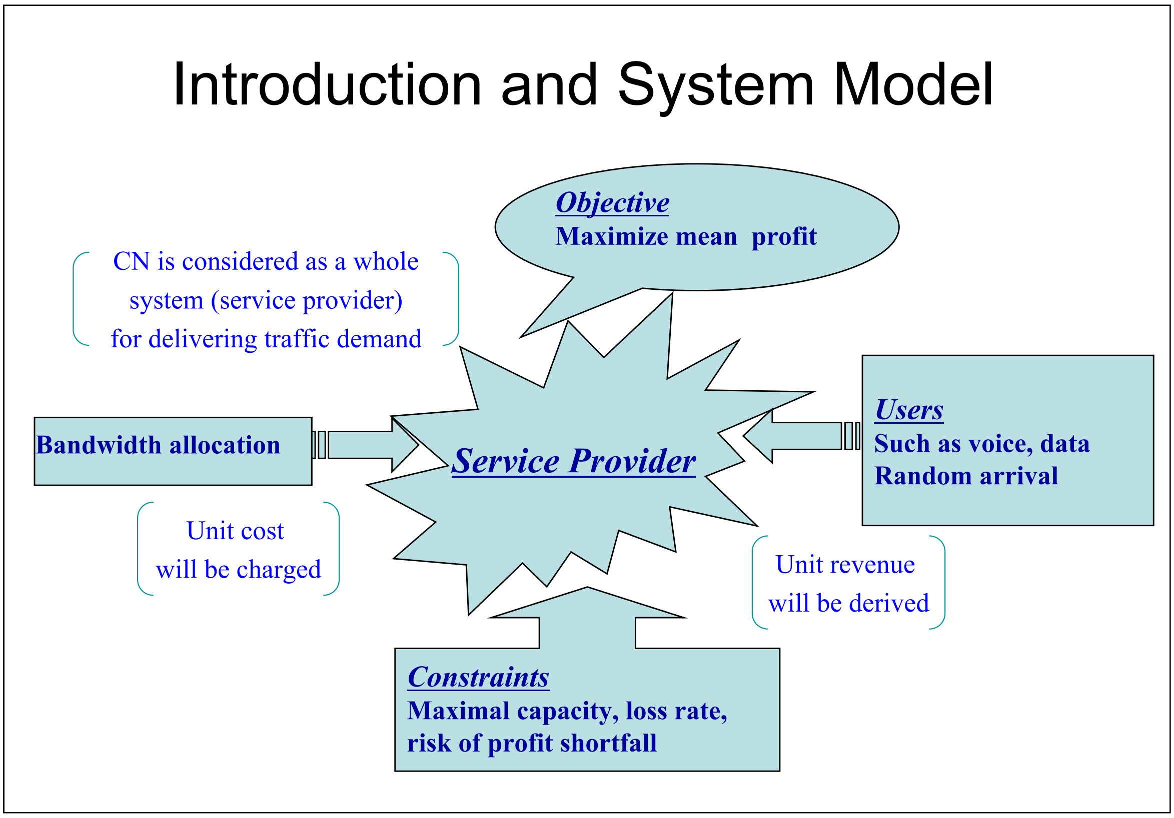
†Jun Wu, Wuyi Yue and †Shouyang Wang

†Academy of Mathematics and Systems Science
Chinese Academy of Sciences, Beijing 100080 China
wujun@iss.ac.cn, sywang@amss.ac.cn

Department of Information Science and Systems Engineering
Konan University, Kobe 658-8501 Japan
yue@konan-u.ac.jp

Abstract

In this paper, we present an analysis of optimization and risk calculation in Communication Networks (CNs). The model is proposed for offline traffic engineering optimization with bandwidth allocation and performance analysis. First, we introduce an optimization model in the CN and derive the optimal bandwidth capacity. Then, we analyze the profit shortfall risk in the CN by using a Conditional Value-at-Risk (CVaR) approach for two typical arrival processes of traffic demand: Poisson arrival process and uniform distribution arrival process. Finally, we give numerical results to show the impact of risk averseness and compare how the characteristics of these two arrival processes of traffic demand affect the network performance.



1. Introduction

Traffic engineering in Communication Networks (CNs) is a process of controlling traffic demand in a network so as to optimize resource utilization and network performance [1], [2]. There are two forms of traffic engineering: online planning and offline planning. Online traffic engineering focuses on instantaneous network states and individual connections. Offline traffic engineering simultaneously examines each channel's resource constraints as well as examining what is needed of each Local Service Provider (LSP) in order to provide global calculations and solutions for the CNs by a centralized view. Traffic engineering has greatly improved network utilization and performance by using emerging technologies such as Multi-Protocol Label Switching (MPLS) and Optical Channel Trails (OCT) [3], [4].

In previous works, the offline optimization problem was formulated as a deterministic Multi-Commodity Flow (MCF) model, where demand of each channel was given as a fixed quantity [5], [6]. The objective usually aims to optimize the network total revenue from serving traffic demand. In the deterministic MCF model, network revenue was assumed to be a linearly increasing function of the amount of bandwidth provisioned up to the optimal capacity, where all traffic demand was satisfied.

Recently, there were some works concerning stochastic traffic engineering. Mitra and Wang presented a stochastic traffic engineering framework for optimizing bandwidth provisioning and path selection in CNs [7]. The objective was to maximize revenue from serving demand in a two-tier market in the telecommunication industry. Mitra and Wang also developed an optimization framework for the network service provider to manage profit in a two-tier market [8]. They investigated the impact of network size and risk averseness on bandwidth management. Mitra and Wang furthered their studies in [7], [8] and developed the efficient frontier of mean revenue and revenue risk [9]. Wu, Yue and Wang proposed a stochastic model for optimizing bandwidth allocation with a loss rate constraint. They analyzed the loss rate constraint and risk averseness in the CN optimization model and showed the impact of loss rate constraint and risk averseness on the network performance [10]. Wu, Yue and Wang also introduced a penalty cost based on the model presented in [10] to guarantee the network performance. They analyzed penalty cost and risk averseness in the CN optimization model and showed the impact of penalty cost and risk averseness on the network performance [11].

Most of the above research papers studied the risk factors associated with performance of CN's by using a mean-variance approach. In this paper, we use a new risk management tool called Conditional Value-at-Risk (CVaR) to study the network risk. This new tool avoids the disadvantage of equally penalizing the desirable upside and the undesirable downside outcomes that is inherent in the mean-variance approach.

The mean-variance analysis, which was first introduced by Markowitz [12], has been a standard tool in risk management. It involves a systematic tradeoff between the mean and the variance [13]. Value-at-Risk (VaR), introduced in 1994, has been extensively used for measuring risk and has become a part of the financial regulations in the world [14]. It allows a manager to specify a confidence level (a certain level of probability) for attaining a certain level of the wealth. Recently, Rockafellar and Uryasev presented an alternative measure of risk with the CVaR approach [15]. It measures the average value of the profit below the given target level. Some empirical evidence proposed by [16] showed that the CVaR approach had superior computational characteristics when it is compared with the VaR approach and the mean-variance approach.

In this paper, we first describe the basic model proposed in [10] and derive the optimal bandwidth capacity without risk. Then, we analyze the system performance and derive the optimal bandwidth capacity by using CVaR approach and show the impact of risk on the network performance by the analysis. We also compare the characteristics of network performance by using CVaR approach presented in this paper with the characteristics of network performance presented in [10], where the mean-variance approach was used. Finally, numerical results are given to show the impact of risk on network performance.

Notations

b : amount of bandwidth provisioned in the CN
r : unit revenue by serving the traffic demand
c : unit cost for unit bandwidth provisioned in the CN
C_{max} : maximal capacity can be allocated in the CN
D : traffic demand in the whole CN with probability density function $f(x)$ and cumulative distribution function $F(x)$
δ : loss rate parameter $(0 \le \delta \le 1)$
ε : confidence level $(0 \le \varepsilon \le 1)$
γ : risk averseness parameter $(0 \le \gamma \le 1)$
u, v : random variables
$u \vee v$: $\max\{u, v\}$
$u \wedge v$: $\min\{u, v\}$
$(u-v)^+$: $\max\{u-v, 0\}$

2. System Model

A Communication Network (CN) is formulated as a collection of nodes and links, that should derive its revenue by delivering traffic load to and from its users. For unit bandwidth capacity allocated to the network, a unit cost will be charged. The objective of this system is to maximize the expected profit of the network. To guarantee the network performance, we introduce and present analyses for the loss rate constraint and the risk of profit shortfall.

Let (*N, L*) denote a CN composed of nodes and links, where *N* is the set of all nodes and *L* is the set of all links in the network. Let *V* denote the set of all node pairs, $v \in V$ denote an arbitrary node pair. Let C_l denote the maximal bandwidth capacity of link *l*, *R*(*v*) denote an admissible route set for $v \in V$, $\xi \in R(v)$ denote the amount of capacity provisioned on route *s*, D_v ($v \in V$) denote the traffic load between node pair $v \in V$, b_v ($v \in V$) denote the amount of bandwidth capacity provisioned between node pair *v*, which can be routed on one or more routes.

In this paper, we consider the CN to be a whole system. We let *b* denote the amount of bandwidth capacity provisioned in the CN. Let *D* denote the traffic demand in the whole CN, which is characterized by a random distribution with its probability density function *f*(*x*) and cumulative distribution function *F*(*x*). $b \wedge D$ is the actual traffic load transmitted in the CN, where "$\wedge$" represents the choice of the smaller value between *b* and *D*. Let *r* denote the unit revenue by serving the traffic demand. Let *c* denotes the unit cost for unit bandwidth capacity allocated in the CN.

Optimal Bandwidth Capacity without Risk

Objective function

$$\max_{b>0}\{\Pi(b,D)\}$$

Constraints

$$P(b \geq \delta \cdot D) \geq 1-\varepsilon$$

$$b \leq C_{\max}$$

where

$$\Pi(b,D) = E[r\min\{b,D\} - cb] = (r-c)b - r\int_0^b F(x)dx$$

3. Optimal Bandwidth Capacity without Risk

3.1 Model formulation

In this section, we present the model for the network bandwidth allocation problem and derive the optimal bandwidth capacity without risk.

The objective function is to maximize the mean profit function derived by the CN with two constraints subject to the objective function. One is loss rate constraint, another is maximal capacity constraint.

The profit function includes revenue derived by serving traffic demand in the CN and the bandwidth allocation cost.

Optimal Bandwidth Capacity without Risk

$$\frac{d^2\Pi(b,D)}{db^2} \le 0 \Longrightarrow \Pi(b,D):\ \text{a concave function of } b$$

$$\frac{d\Pi(b,D)}{db} = 0 \xrightarrow{\text{First order condition}} b = F^{-1}\left(\frac{r-c}{r}\right)$$

Constraints

$$P(b \ge \delta \cdot D) \ge 1-\varepsilon \Rightarrow b \in [\delta \cdot F^{-1}(1-\varepsilon), \infty)$$

$$b \le C_{\max}$$

Optimal solution

$$b^* = [F^{-1}\left(\frac{r-c}{r}\right) \vee \delta \cdot F^{-1}(1-\varepsilon)] \wedge C_{\max}$$

3.2 Optimal solution

With the above preparation, we can derive the optimal bandwidth capacity that should be allocated in the CN.

First, we analyze the property of the mean profit function without any constraints. We can derive the first order derivative and second order derivative functions respectively. From the assumptions, we know that the mean profit function is a concave function of the bandwidth capacity *b*.

Second, we consider the loss rate constraint and the maximal capacity constraint.

Finally, we can get the optimal bandwidth capacity with constraints.

Risk Analysis with Conditional Value-at-Risk (CVaR) Approach

Objective function

$$\max_{b>0} C_\gamma \{\pi(b,D)\}$$

with $C_\gamma(\pi(b,D)) = \max_{v \in R} \{v + \tfrac{1}{\gamma} E[(\pi(b,D) - v)^-]\}$

$$\pi(b,D) = r(b \wedge D) - cb$$

(): negative part of the term in brakets

Objective function can be written as follows

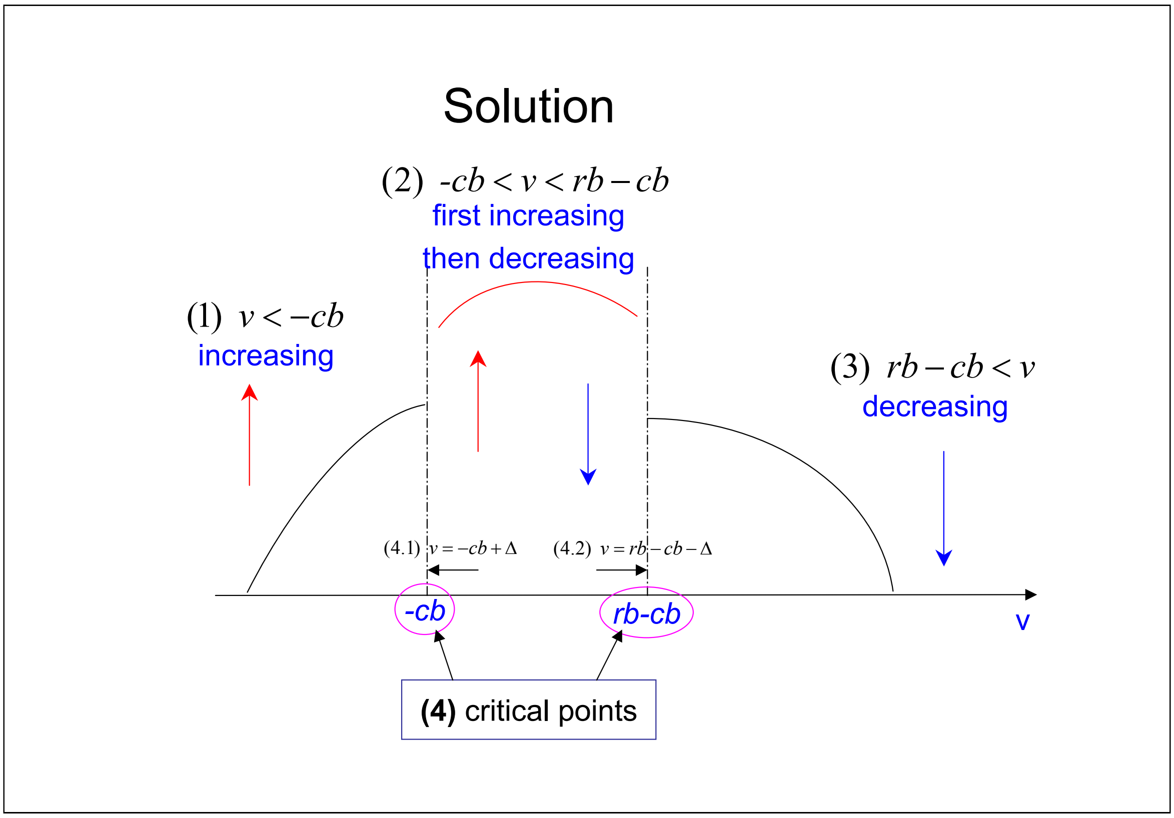
$$\max_{b>0} \max_{v \in R} \{g(v,b)\}$$

$$g(v,b) = \{v + \tfrac{1}{\gamma} E[(\pi(b,D) - v)^-]\}$$

4. Risk Analysis in Communication Networks

The risk analysis in this section is based on the concept of Conditional Value-at-Risk (CVaR), which was proposed in financial risk management. In the following, we briefly review the literature of this stream.

The mean-variance analysis, which was first introduced by Markowitz [12], had been a standard tool in risk management. It involves a systematic tradeoff between the mean and the variance [13]. Value-at-Risk (VaR), introduced in 1994, had been extensively used for measuring risk and became part of the financial regulations [14]. It allows a manager to specify a confidence level for attaining a certain level of the wealth. Recently, Rockafellar and Uryasev presented an alternative measure of risk with the name of CVaR [15]. It measures the average value of the profit below the given quantile level. Some empirical evidence proposed by [16] showed that the CVaR approach had good computational characteristics over VaR approach and mean-variance approach. In this paper, we study the risk in CNs based on CVaR approach.



According to the objective function, we know that to find the optimal bandwidth capacity is equivalent to a two-step optimization: first is to optimize *v*, second to optimize *b*.

With respect to the first-step optimization, we distinguish four cases to derive the optimal solution *v**.

(1) *v* < *-cb*;

(2) *-cb* < *v* < *rb-cb*;

(3) *rb-cb* < *v*;

(4) when v approaches *-cb* from the right, and also when *v* approaches *rb-cb* from the left.

With respect to the second-step optimization, we distinguish two cases to derive the optimal solution *b**.

(1) $b = F^{-1}(\gamma)$

(2) $b < F^{-1}(\gamma)$

From the above analysis, we can get the optimal bandwidth capacity is given as follows: $b^* = F^{-1}(\gamma \frac{r-c}{r}).$

Numerical Results

Comparing with the model without risk, the results presented in this paper enlarge the dimension of the problem and provides more insights for network managers with different risk preference

When the parameter $\gamma = 1$, the result of the model with risk in this paper is the same as that of the model without risk
When the parameter $\gamma = 0$, it means that no bandwidth will be allocated in the CN, i.e., the network manager is unwilling to provide any services
When the parameter $0 < \gamma < 1$, the optimal bandwidth obtained in this paper is less than the optimal bandwidth without risk

Comparing with the results by mean-variance analysis presented in Wu, Yue & Wang (2005), the results in this paper reveal advantages of using CVaR approach over mean-variance approach

It avoids the disadvantage of the mean-variance approach which equally penalizes the desirable upside and the undesirable downside outcomes
It also provides a closed-form solution, which has good computational characteristics over the mean-variance approach

4.2 Remarks

Remark 1:

Comparing with the model without risk presented in [10], the result presented in this paper enlarges the dimension of the problem without risk and provides more insights for network managers with different risk preference.

Remark 2:

Comparing with the mean-variance analysis model presented in [10], the result presented in this paper reveals advantages of using CVaR approach over the mean-variance approach. It avoids the disadvantage of the mean-variance approach which equally penalizes the desirable upside and the undesirable downside outcomes. It also provides a closed-form solution, which has good computational characteristics over the mean-variance approach.

In the following, we will present two typical arrival processes to express the arrival of traffic demand in a CN. One is Poisson arrival process, another is process with uniform distribution.

Poisson arrival process

Mean profit function

$$\Pi^*(b,D)=\frac{r}{\lambda}(1-e^{-\lambda b^*})-cb^*$$

Optimal bandwidth capacity

$$b^*=-\frac{Ln[1-\gamma\frac{r-c}{r}]}{\lambda}$$

$Ln[\cdot]$: natural logarithm function to base e.

Process with uniform distribution

Mean profit function

$$\Pi^*(b,D)=-\frac{r}{2}b^2+(r-c)b$$

Optimal bandwidth capacity

$$b^*=\gamma\frac{r-c}{r}$$

According to the engineering experience, we choose several different arrival rates to represent the different cases of traffic load in the CN:

$\lambda=0.1$: the case that the traffic load is low

$\lambda=0.5$: the case that the traffic load is normal

$\lambda=0.9$: the case that the traffic load is heavy

4.3 Special cases analyses

In this section, we will present two typical arrival processes to express the arrival of traffic demand in a CN. One is Poisson arrival process, another is process with uniform distribution.

First, we consider a fully distributed communication network, where the traffic demand offered to the whole CN forms a Poisson process, since in most of the system models in CNs, the arrival process of traffic demand is assumed to form a Poisson process.

Second, we consider the same fully distributed communication network, but the traffic demand offered to the whole CN forms a uniform distribution U[*m*, *n*] on some interval [*m*, *n*], since in some of the system models of CNs (such as ATM system), the arrival process of traffic demand is assumed to form a uniform distribution.

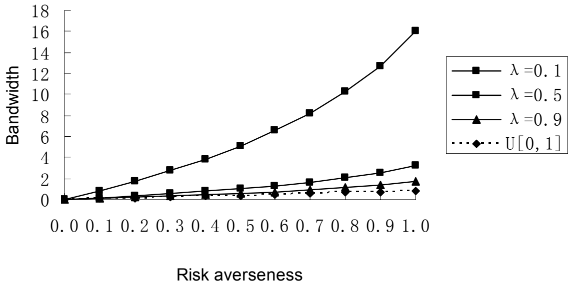


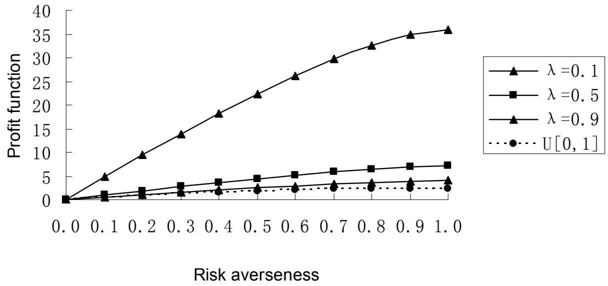
Figure 1. Impact of risk on bandwidth capacity.

- In all curves, the optimal bandwidth with risk is always less than the optimal bandwidth without risk and the bandwidth increases as the risk decrease
- The curve with a smaller arrival rate has a quicker increasing speed than the curve with a larger arrival rate
- With the same risk averseness, the heavier the traffic load is, the less the impact of the risk on the bandwidth capacity will be
- With the same risk averseness, all the curves with the assumption of exponential distribution reveal a larger impact than the curve with the assumption of uniform distribution

The horizontal axis γ of Fig. 1 corresponds to the decrease in risk averseness. The ordinate axis b^* of Fig. 1 corresponds to the bandwidth capacity. We set the risk parameter γ to increase from 0.0 to 1.0 by 0.1 each step with all other parameters unchanged.

Our numerical results include the optimal bandwidth capacity obtained without risk presented in [10], which is one point in the ordinate axis corresponding to γ=1.0 in Fig. 1.

Compared with the results presented in [10] without risk, the numerical results in our paper reveal a distinct impact of risk on the network bandwidth capacity.



Figures 2. Impact of risk on mean profit.

- In all curves, the mean profits with risk are always less than the optimal profit without risk and the profit increases as the risk decreases
- The curve with a smaller arrival rate has a quicker increasing speed than the curve with a larger arrival rate
- With the same risk averseness, the heavier the traffic load is, the less the impact of the risk on the profit will be
- With the same risk averseness, all the curves with the assumption of exponential distribution reveal a larger impact than the curve with the assumption of uniform distribution

The horizontal axis γ of Fig. 2 corresponds to the decrease in risk averseness. The ordinate axis *(b D) of Fig. 2 corresponds to the mean profit. We set the risk parameter γ to increase from 0.0 to 1.0 by 0.1 each step with all other parameters unchanged.

Our numerical results include the optimal profit obtained without risk presented in [10], which is one point in the ordinate axis corresponding to

γ=1.0 in Fig. 2.

Compared with the results presented in [10] without risk, the numerical results in our paper reveal a distinct impact of risk on the network profit function.

Conclusions

- We have analyzed the risk averseness in the CNs by using Conditional Value-at-Risk (CVaR) approach and compare the characters of CVaR with the mean-variance approach
- We have given numerical results to compare our model with the previous model and shown the impact of risk on the network performance

- Risk averseness has distinct impact on the network performance
- Implications presented in this paper have insights for traffic engineering design and planning

6. Conclusions

In this paper, we presented a stochastic model for bandwidth allocation and performance analysis in Communication Networks (CNs) with risk analysis. We have derived the optimal bandwidth allocation capacity with risk averseness. We have analyzed the risk averseness in the CNs by using the Conditional Value-at-Risk approach. We have given numerical results to compare our model with the previous model presented in [10] and shown the impact of the risk on the network performance for two arrival processes of traffic demand. We can conclude that risk averseness has distinct impact on the network performance. The implications presented in this paper have insights for traffic engineering design and planning.

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