Multi-stage Stackelberg Game Approach for Colocation Datacenter Demand Response

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The 19th Asia-Pacific Network Operations and Management Symposium

APNOMS 2017

September 27-29, 2017, Seoul, Korea

== Managing a World of Things ==

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September 28th, 2017

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Outline

- Introduction
- System Model
- CDC Operator Decision
- CSP Decision
- Numerical Analysis
- Conclusion

Introduction: Key Concepts

- **Colocation Datacenters (CDCs)**
  - CDC houses multiple tenants who individually manage servers in a shared building.
  - The building operator supports the facility (e.g., power, cooling, etc.) without any control over tenants IT systems.
- **Demand Response (DR)**
  - **Mandatory** *emergency* DR (inelastic)
  - **Voluntary** *economic* DR (elastic)
- **Curtailment Service Provider (CSP)**
  - Demand Response Provider role
  - An authorized mediator between independent system operator (ISO) and the CDCs customers for participating in the DR program.

Introduction

Demand Response of Datacenters

- Most works on DR of DC studied the ancillary and/or emergency DR.
- Economic DR of CDCs and CSP:
  - CDC operator has no control over tenants' IT systems, (e.g. servers)
  - Flexible control on an elastic demand response capacity in that they can *at will* reduce the electricity usage for payment based on *dynamic price signals*.

➢ We fill this gap with an incentive mechanism for CDC economic DR under the CSP control

❑ Demand Response of CDCs

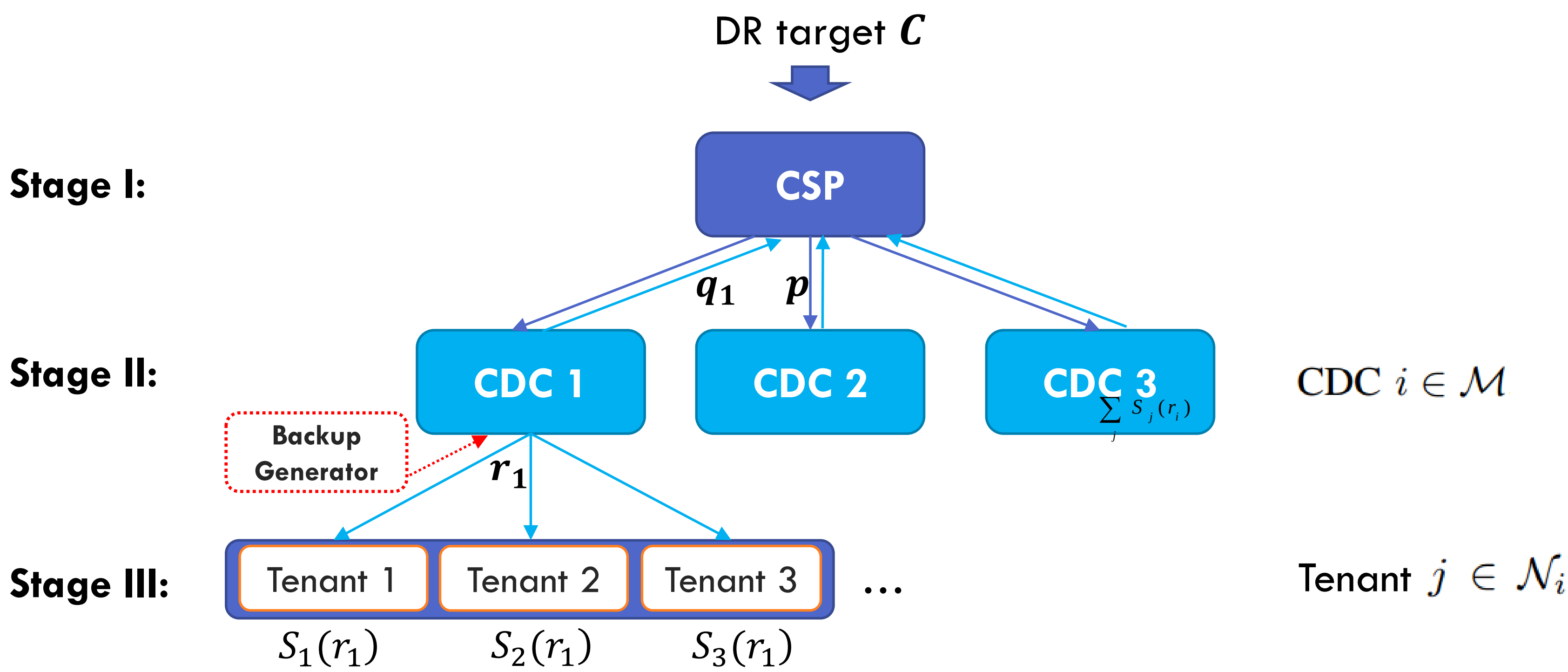
- CDCs could be a common solution to a variety of Internet content service providers, cloud service providers.
- 1685 CDCs in the U.S.
- CDC market is forecast to reach $43 billion in 2018
- CDCs consume 37% of electricity of DCs power usages in the U.S.

http://www.datacentermap.com/usa/datacenters.html

❑ Our Contributions

- We design an incentive mechanism to incentivize CDC to curtail energy consumption for economic DR.
- Strategic decisions of CSP, CDCs and tenants and their interactions are modeled as a **three-stage Stackelberg game.**
- In **Stage I**, we design an efficient algorithm by reducing the search space of the CSP's optimal pricing problem.
- In **Stage II**, we design a near-optimal approximate approach that only requires limited information to find the optimal procurement and reward strategy for each CDC.
- In **Stage III,** the optimal tenants' energy reduction is calculated.
- We perform an extensive numerical analysis to compare the individual CSP cost with social CSP cost.

System Model



➢ Backup generator can be used to supplement its response deficit:
- $q_i - \sum_j S_j(r_i)$

❑ Tenant Model in Stage III

- Cost function (e.g., wear-and-tear, performance degradation, etc.)

$$V_j(e_j) = \omega_j \, e_j^{\alpha_i},$$

  - ω_j is the unit cost (history), all tenants of CDC i have the same *sensitivity* parameter α_i.

- Given reward r_i, a rational tenant j will decide the optimal reduced energy

$$\max_{e_j \geq 0} \; u_j(e_j) \; = \; r_i e_j \; - \; V_j(e_j).$$

- The unique tenant supply j :

$$S_j(r_i) := e_j^* = V_j'^{-1}(r_i) = \left(\frac{r_i}{\omega_j}\right)^{\frac{1}{\alpha_i - 1}} \alpha_i^{\frac{1}{1-\alpha_i}} = \omega_j^{\frac{1}{1-\alpha_i}} s_i(r_i) \; = \bar{\omega}_j s_i(r_i).$$

where $\bar{\omega}_j := \omega_j^{\frac{1}{1-\alpha_i}}, \; s_i(r_i) := (r_i/\alpha_i)^{\frac{1}{\alpha_i - 1}}.$

↓

i.i.d. R.V., $\forall j \in \mathcal{N}_i$
(**private**)

- The aggregate supply of CDC i with the R.V. $X_i := \sum_{j=1}^{N_i} \bar{\omega}_j.$

$$\sum_{j=1}^{N_i} S_j(r_i) = X_i \, s_i(r_i).$$

System Model

❑ CDC Model in Stage II

- Given p, each CDC will maximize its expected profit

$$\mathbf{P_1}: \max_{q_i, r_i \geq 0} \quad \Pi_i^{op}(q_i, r_i; p) := \mathbf{E}\left[pq_i - \left(r_i \sum_{j=1}^{N_i} S_j(r_i) + \gamma \; [D_i]^+ \right) \right] \tag{7}$$

$$\text{s.t.} \quad D_i = q_i - \sum_{j=1}^{N_i} S_j(r_i), \tag{8}$$

- The profit includes revenue, the incentive cost, and the backup power cost
- Decide the energy reduction response q_i and reward r_i
- Response deficit D_i is a R.V.
- Given p, a profile $(r_i^*(p), q_i^*(p))_{i \in \mathcal{M}}$ is an optimal strategy profile
  - **Competitive equilibrium** satisfies the following condition:

$$\Pi_i^{op}(q_i^*, r_i^*; p) \geq \Pi_i^{op}(\bar{q}_i, \bar{r}_i; p), \forall i \in \mathcal{M}, \bar{q}_i, \bar{r}_i \geq 0. \tag{9}$$

For shortage, we ignore the price p in $r_i^*(p), q_i^*(p)$.

System Model

❑ CSP Model in Stage I

- CSP minimizes the DR cost

$$\mathbf{P_2}: \quad \min_{(p,c)\geq 0} \quad \Pi^{lse}(p,c) := p\sum_{i=1}^{M} q_i(p) + \lambda\left(C-c\right)^2$$
$$\text{s.t.} \quad \sum_{i=1}^{M} q_i(p) = c. \qquad (10)$$

- DR cost includes the payment for CDCs and a penalty of the deviation from the target C
- The quadratic penalty function to reflect the CSP use of alternative energy sources to bridge the gap $(C - c)$.
    - A soft target (elastic)

CDC Operator Decision

- As the perspective of the classical news-vendor problem [1]:
  - *(a) maximization of the expected risk-less-profit*
  - *(b) minimization of the expected loss due to the uncertainty of X_i.*
- Introduce an auxiliary variable $z_i := q_i / s_i(r_i)$

Theorem 1. *There exists a unique optimum $(q_i^*, r_i^*)_{i \in \mathcal{M}}$ at Stage II of the Stackelberg game with a given $p \geq 0$ such that*

$$z_i^* = \begin{cases} F_{X_i}^{-1}(1) = X_i^u, & \text{if } p \geq \gamma, \\ F_{X_i}^{-1}\left(\frac{p}{\gamma}\right), & \text{if } p \leq \gamma, \end{cases}$$
$$r_i^* = \begin{cases} \frac{\gamma}{\alpha_i}, & \text{if } p \geq \gamma, \\ \frac{\gamma}{\alpha_i} \cdot \frac{\int_0^{p/\gamma} F_{X_i}^{-1}(w)\, dw}{\mathbf{E}[X_i]}, & \text{if } p \leq \gamma, \end{cases} \tag{14}$$

where $F_{X_i}^{-1}$ is the quantile function of F_{X_i}. As a result, $q_i^* = z_i^* \, s_i(r_i^*)$.

Corollary 1. *If $p \leq \gamma$, according to CLT* ➡ ***Approximation***

$$z_i^* = \Phi^{-1}(p/\gamma)\sqrt{N_i}\sigma_i + N_i\mu_i, \tag{15}$$
$$r_i^* = \frac{\gamma}{\alpha_i} \cdot \frac{\int_{x_{lo}}^{x_{up}} \left(\sqrt{N_i}\sigma_i\, x + N_i\mu_i\right) d\Phi(x)}{N_i\, \mu_i}, \tag{16}$$
$$q_i^* = z_i^* \, s_i(r_i^*), \tag{17}$$

where $x_{lo} = \frac{X_i^{lo} - N_i\mu_i}{\sqrt{N_i}\sigma_i}$, $x_{up} = \Phi^{-1}(p/\gamma)$, and $\Phi^{-1}(\cdot)$ is the quantile of a standard normal distribution.

[1] N. C. Petruzzi and M. Dada, "Pricing and the news vendor problem: A review with extensions," *Operations Research*, vol. 47, no. 2, pp.183–194, February 1999.

CSP Decision

- Operator response has the following form:

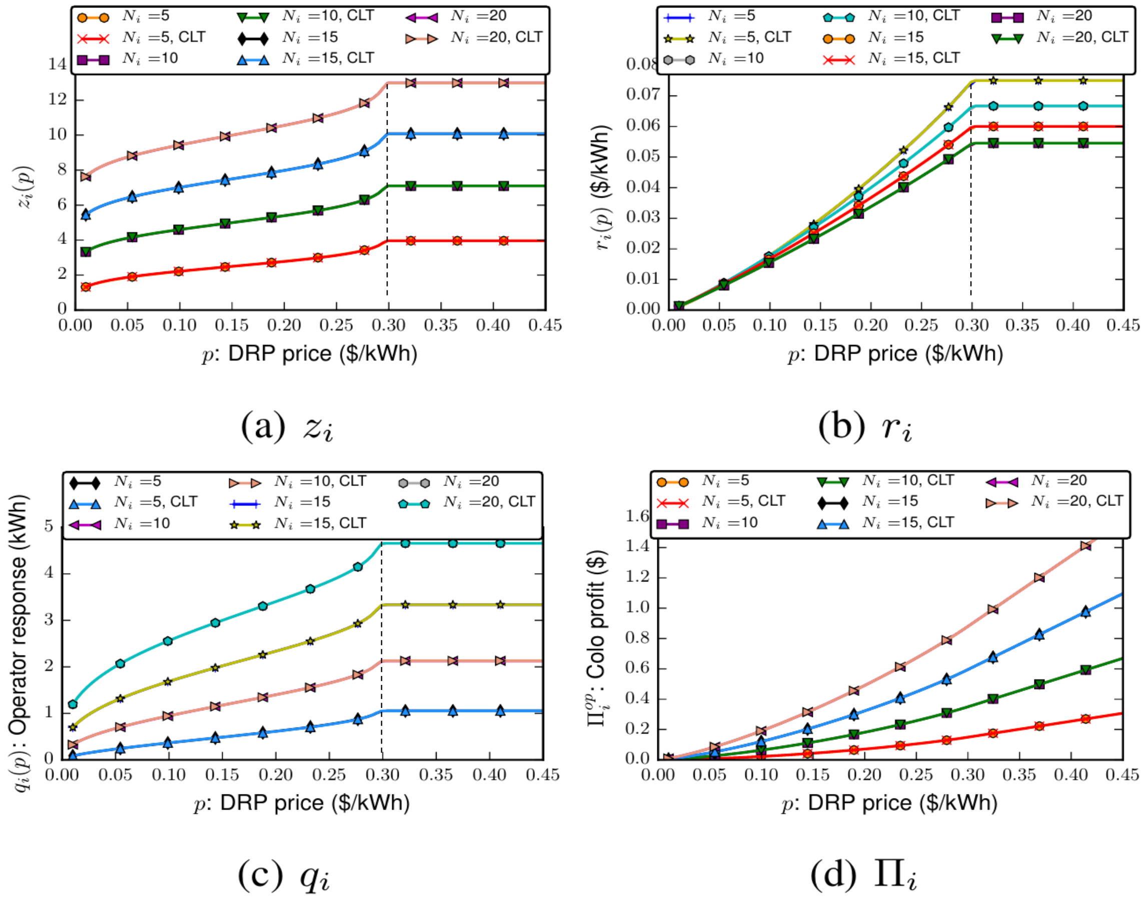
$$q_i^*(p) = \begin{cases} q_i^{\max} := F_{X_i}^{-1}(1)\left(\frac{\gamma}{\alpha_i^2}\right)^{\frac{1}{\alpha_i - 1}}, & \text{if } p \geq \gamma; \\ \left(\frac{\gamma}{\alpha_i^2 \mathbf{E}[X_i]} \int_0^{p/\gamma} F_{X_i}^{-1}(x)\, dw\right)^{\frac{1}{\alpha_i - 1}} F_{X_i}^{-1}\left(\frac{p}{\gamma}\right), & \text{if } p \leq \gamma. \end{cases}$$

- Then, the CSP's problem **P2** can be rewritten as follows

$$\begin{aligned} \min_{p \geq 0}. \quad & \sum\nolimits_{i=1}^{M} p\, q_i^*(p) + \lambda\big(C - c\big)^2 \\ \text{s.t.} \quad & \sum\nolimits_{i=1}^{M} q_i^*(p) = c. \end{aligned} \qquad (19)$$

Numerical Analysis

- **CDCs Performance:**
  - CDC has $\alpha_i = 2$, backup generator $0.3\ \$/\mathrm{kWh}$
  - $\omega_j, \varpi_j \in Unif(0,1)\ (\$/\mathrm{kWh})$
  - X_i follows Irwin-Hall distribution



(a) z_i (b) r_i

(c) q_i (d) Π_i

Fig. 1: CDC operator parameters with different N_i.

Numerical Analysis

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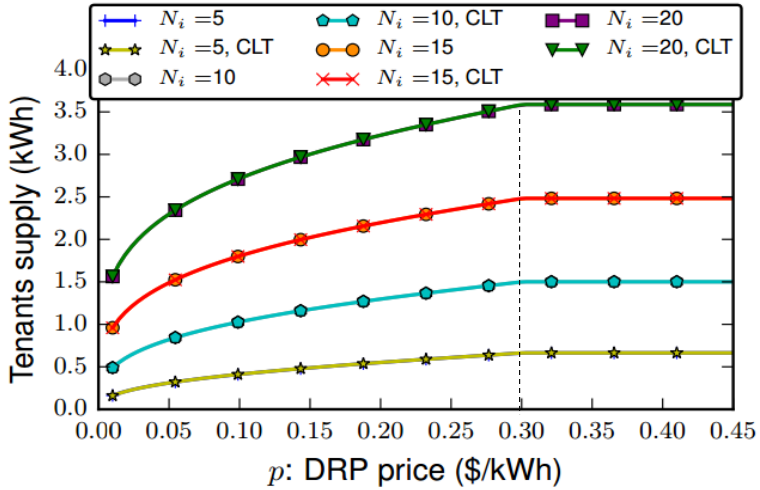
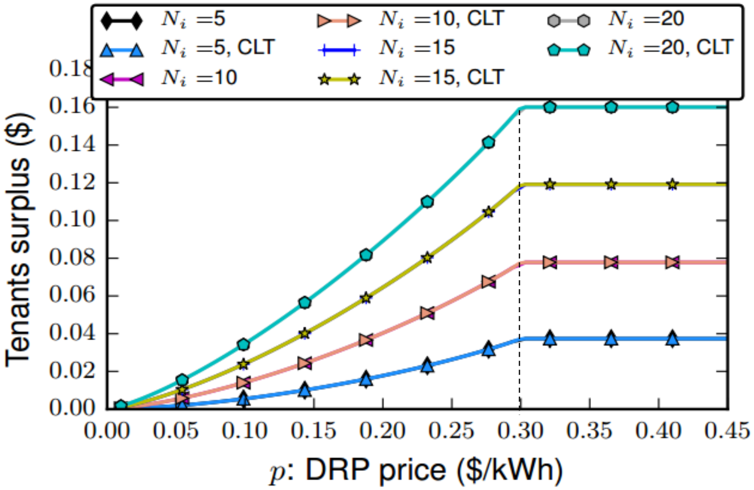


(a) Tenant response.



(b) Tenant surplus.

Fig. 2: Tenant performance.

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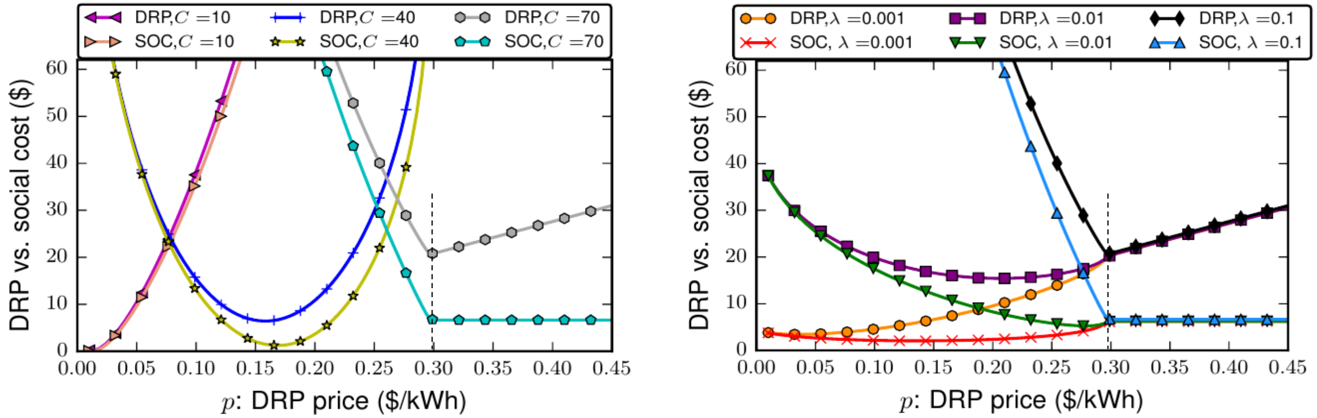
- **CSP Performance:**
  - CSP has 8 CDCs.
  - Benchmark - Optimal social cost:

$$\mathbf{P_3}: \min_{p,c\geq 0} \quad \Pi^{soc}(p,c)$$
$$\text{s.t.} \quad \sum_{i=1}^{M} q_i(p) = c, \tag{20}$$

where $\Pi^{soc}(p,c)$ is defined to be

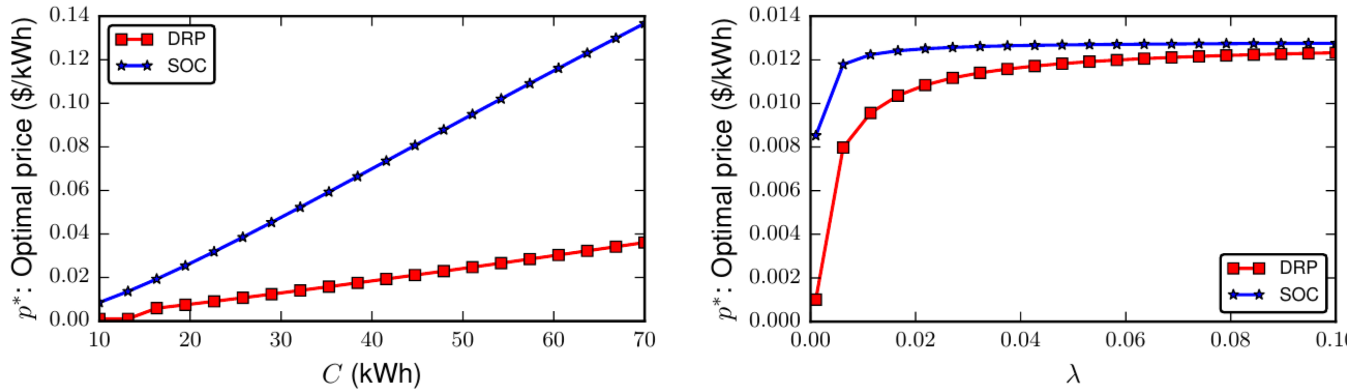
$$\sum_{i=1}^{M}\sum_{j=1}^{N_i}\mathbf{E}\left[V_j(S_j(r_i)) + \gamma\Big[q_i - \sum_{j=1}^{N_i} S_j(r_i)\Big]^+ + \lambda(C-c)^2\right], \tag{21}$$

Numerical Analysis



(a) Fix λ, vary C. (b) Fix C, vary λ.

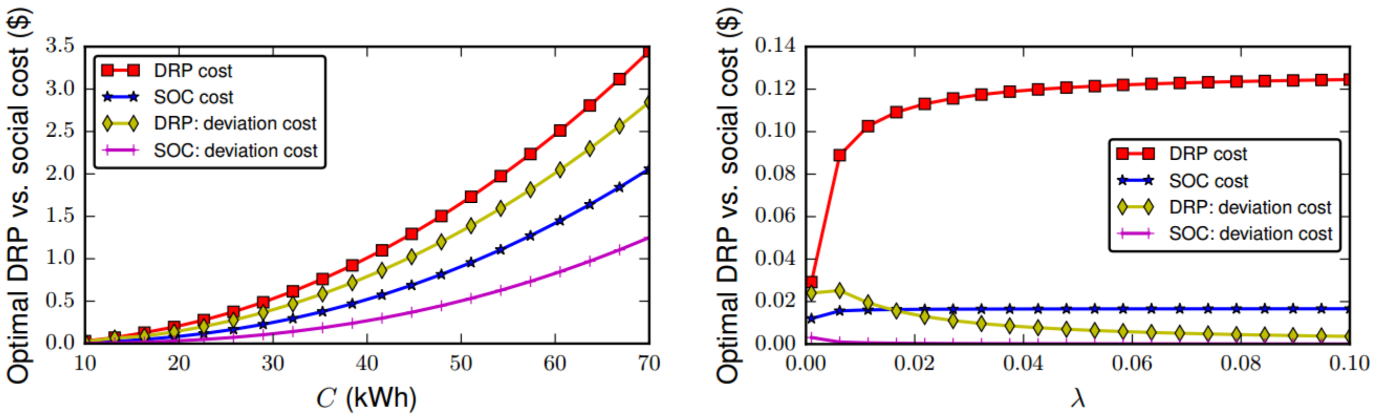
***Fig. 3:** Comparison of CSP and social costs with different prices.*



(a) Increasing C. (b) Increasing λ.

***Fig. 4:** Comparison of optimal CSP compensation price with social price.*

Numerical Analysis



(a) Increasing C.

(b) Increasing λ.

Fig. 5: *Comparison of optimal CSP cost with social cost.*

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Conclusion

- Our work designs an incentive mechanism for CDC economic DR by considering the role of CSP.
- The strategic interactions between CSP, CDCs and tenants are formulated as a three-stage Stackelberg game.
- The CSP determine its compensation price, then CDCs compute the reward for tenants and an amount of the energy reduction.
- We also showed that a Stackelberg equilibrium exists where CSP, CDCs and tenants make optimal decisions.
- We provide extensive numerical study and presented the comparison of the optimal CSP compensation cost with the social cost.

Question & Answer

Thank You !!!